

M-200 Errors, Uncertainties, and Excel

OBJECTIVES:

1. To reassure you that nature is inherently uncertain. All measurements include some degree of uncertainty.
2. To practice the fundamentals of characterization of uncertainty and propagation of uncertainty, using Excel for calculations and plotting.
3. To develop a vocabulary for rigorous and objective reporting of measurements and their relation to predictions.

APPARATUS:

Funnel; 100 round chads from a three-hole punch, graph paper; Excel.

PRELAB:

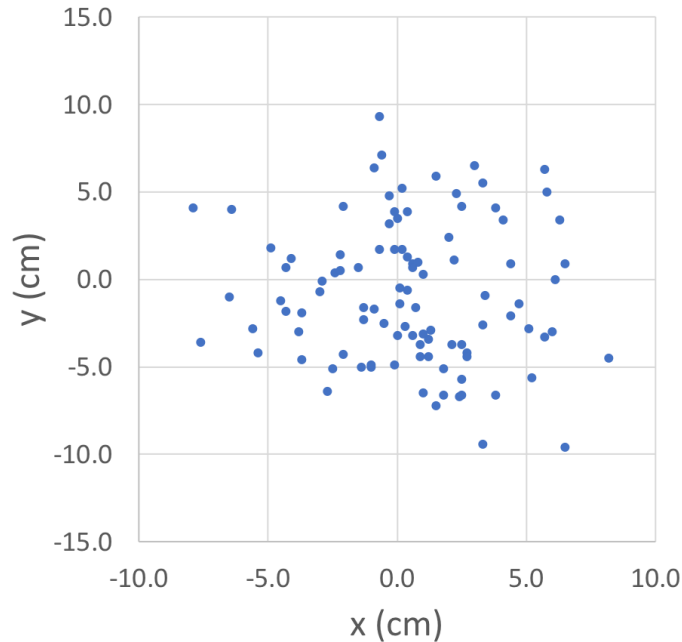
1. Read through the “Errors and Uncertainties” section, beginning on page 3.
2. Find documentation on the web for the Excel functions AVERAGE, STDEV, and LINEST that pertains to your version of Excel.
3. Familiarize yourself with how to make a scatter chart (without connecting lines) of data in Excel, how to add error bars to the data points, and how to add a trendline.

PROCEDURE

1. Set the bottom of the funnel 10 cm above the graph paper. Set the center of the graph paper directly below the center of the funnel, and mark the origin of a set of x - y axes on the graph paper directly below the center of the funnel. Indicate x and y directions.
2. Pour the 100 chads through the funnel. Make a mark on the graph paper where the center of each chad has come to rest. Transfer data to Excel, indicating x and y coordinates of each of the chad centers.
3. Repeat for four other heights of the funnel: 15 cm, 20 cm, 25 cm, and 30 cm.

MEAN AND STANDARD DEVIATION

1. Use Excel to make a scatter chart of the $h=25$ cm data. It should look like the figure below.



2. Calculate the mean $\bar{x}$ (using AVERAGE) and standard deviation σ_x (using STDEV) of the x coordinates of the data.
3. Calculate $\bar{y}$ and σ_y from the y coordinates of the data.
4. Take a piece of paper and start a sketch of the scatter chart you just made by drawing just the axes. Make sure the x -range and y -range are the same. Don't add the dots yet.
5. Draw vertical lines on the paper at the locations $\bar{x}$, $\bar{x} + 0.6745\sigma_x$, and $\bar{x} - 0.6745\sigma_x$.
6. Draw horizontal lines on the paper at the locations $\bar{y}$, $\bar{y} + 0.6745\sigma_y$, and $\bar{y} - 0.6745\sigma_y$.
7. We will assume that the distributions of the x - and y - positions of the dots are "normal distributions", and therefore obey the following rules:
 - Half the dots are on one side of the mean, and half are on the other;
 - Half the dots are within 0.6745 standard deviations of the mean.
8. Following the above rules, add 100 dots to your sketch.
9. Make a sketch of 30 dots that have mean positions $\bar{x} = 2$ cm, $\bar{y}=0$ cm, $\sigma_x = \sigma_y = 3$ cm.
10. Make a sketch of 30 dots that have mean positions $\bar{x} = 0$ cm, $\bar{y}=2$ cm, $\sigma_x = 1$ cm, $\sigma_y= 5$ cm.

STANDARD DEVIATION OF THE MEAN

The standard deviation of the mean σ_μ is a measure of the uncertainty of the mean. The larger the number N of data points you have, the smaller σ_μ becomes: $\sigma_\mu = \sigma/\sqrt{N}$. The standard deviation of the mean σ_μ is different from the standard deviation σ discussed in the last section, which is a measure of the scatter of the data.

1. Calculate the standard deviation of the mean $\sigma_{\mu,x}$ for the x -position data for $h=25$ cm.
2. Calculate the standard deviation of the mean $\sigma_{\mu,y}$ for the y -position data for $h=25$ cm.
3. Return to the sketch you made in step 4 above, and draw a particularly dark dot at the position of $(\bar{x}, \bar{y})$.
4. Add vertical error bars to the dark dot that extend $0.6745 \sigma_{\mu,y}$ above it and $0.6745 \sigma_{\mu,y}$ below it.
5. Add horizontal error bars to the dark dot that extend $0.6745 \sigma_{\mu,x}$ to either side of it.

Note that the error bars are much smaller than the “scatter” of the 100 dots, which is characterized by the standard deviations σ_x and σ_y .

The error bars convey that if someone else were to repeat the experiment of dropping 100 chads through an identical funnel, that person should expect a 50% chance that the center of the distribution of the chad locations would lie within the error bars.

PROPAGATION OF UNCERTAINTIES

The center of the distribution of dots is located at $(\bar{x}, \bar{y})$, and from this we can calculate $\bar{r} = \sqrt{\bar{x}^2 + \bar{y}^2}$, the distance of the center of the distribution from origin (this is a quite different distance than the answer to the question “how far on average did a chad travel as it fell?”). The question then arises, “what is the uncertainty in $\bar{r}$?”

The answer can be worked out using multi-variate calculus (not expected of students in Physics 201 or 207):

$$\sigma_r = \frac{\sqrt{(\bar{x} \sigma_{\mu,x})^2 + (\bar{y} \sigma_{\mu,y})^2}}{\bar{r}}. \quad (1)$$

1. Calculate $\bar{r}$ and σ_r for the $h=25$ cm data.
2. Calculate $\bar{r}$ and σ_r for the other four sets of data.

You have just asked Excel to calculate the uncertainty in a quantity you did not directly measure ($\bar{r}$) based on uncertainties in quantities you did directly measure (x and y). This is an example of the “propagation of uncertainties”.

FITS AND UNCERTAINTIES IN FIT PARAMETERS

It looks like the scatter of the chads, as well as the distance from the center of the distribution of chads to the origin, increases with increasing h . Let us suppose this increase to be linear in h , and compare this supposition to the measurements.

1. In Excel, make a scatter chart (without connecting lines) of $\bar{r}$, the distance of the center of the distribution of chads from the origin, as a function of h .
2. Add a linear trendline to the chart. Note that what Excel calls a “trendline” is what in physics is called a “least-squares fit”. Select “Display equation on chart”.
3. Excel also calculates the uncertainties in the fit parameters. They can be accessed through the Excel function LINEST. See cells V6:W10 (raw data) and cells V13:W14 (trendline fit parameters and uncertainties) for an example.
4. Use LINEST to display the uncertainties in the trendline found in step 2.
5. By hand, make a copy on paper of the scatter chart and trendline from steps 1 and 2. Add vertical error bars to each data point, extending above and below the data point by $0.6745 \sigma_r$.

Excel has quite a bit more capacity for analysis of uncertainties than we will use in Physics 201 or 207.

VOCABULARY: “DISCREPANCY”

discrepancy (noun) an unexplained difference

If we lower the funnel until it sits on the paper ($h = 0$), every chad will land exactly at the origin ($r = 0$). However the linear trendline found in step 2 above does not go through the point $h = 0, r = 0$. There is some kind of discrepancy (unexplained difference) between this new data point and the linear trendline.

1. What is the discrepancy (in cm) between $r = 0$ and the value of the trendline at $h = 0$? (In physics 201 always report discrepancies as being greater than zero.)
2. What is the ratio R of (the discrepancy)/(the uncertainty in the trendline value at $h = 0$) ? (R should be a dimensionless number greater than zero.)
3. Summarize this result by writing the sentence: “The discrepancy between the observed r at $h = 0$ and the value of the linear trendline at $h = 0$ is R times as big as the uncertainty in the trendline value”. This elaborate phrasing is objective—it does not express an opinion on whether the observation or the trendline (or both) are in error. Also it is rigorous: it proceeds from the raw data by a set of automatic mathematical steps.

SUMMARY

1. Summarize what you did in this lab, and tell us what part of the material was new (this will help us teach you better).
2. What version of Excel are you using?